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THE ANALYSIS OF THE IMPACT OF SELECTED VEHICLE PARAMETERS ON THE LIMIT FORCES ON THE VEHICLE'S DRIVE WHEELS

ANALIZA WPŁYWU WYBRANYCH PARAMETRÓW POJAZDU NA GRANICZNE SIŁY NA KOŁACH JEZDNYCH POJAZD

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Summary

Stability during vehicle movement is considered an exceptionally important aspect. It is particularly important from the point of view of active safety in modern vehicles. In this regard, it is significant to know the maximum forces that occur at the contact of the wheels with the roadway. This article attempts to determine the value of these forces. For this purpose, a calculation algorithm was developed, using the Dynamic Square Method (DSM). This is a method for determining the limit force value on vehicle wheels. For the purpose of the analysis a two-wheel vehicle model was used. With its application, the characteristic of the longitudinal limit forces on the wheels was calculated, depending on the lateral acceleration values for selected vehicle reference characteristics. The DSM method was also used to analyze the impact of parameter changes such as the height of the center of gravity and axle mass distribution on the values of these longitudinal thrust forces on the wheels. As a result of the implementation of the DSM algorithm, the characteristics for a quadrangle bounded field are obtained, presented as a set of isoclines of constant lateral acceleration values achieved for the given longitudinal forces on the front and rear axle wheels. By analyzing the obtained graphs, it is possible to determine the effect that the change of the given parameter has on the achieved longitudinal force values.

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Keywords: vehicle dynamics, longitudinal and lateral forces, friction forces, longitudinal and lateral acceleration, Dynamic Square Method

Streszczenie

Zachowanie stateczności podczas ruchu pojazdu jest bardzo ważnym aspektem. Ma szczególne znaczenie z punktu widzenia bezpieczeństwa czynnego we współczesnych pojazdach. Istotnym zagadnieniem jest przy tym znajomość maksymalnych sił, jakie występują na styku kół z jezdnią. W niniejszym artykule podjęto próbę wyznaczenia wartości tych sił. Wykorzystano do tego algorytm obliczeniowy opracowany przy użyciu metody Dynamic Square Method (DSM). Jest to metoda służąca do określania granicznych sił na kołach pojazdu. Do analizy wykorzystano dwukołowy model pojazdu. Przy jego użyciu sporządzono charakterystykę granicznych sił wzdłużnych na kołach w zależności od przyspieszenia poprzecznego dla konkretnych danych pojazdu wzorcowego. Metodę DSM wykorzystano również do analizy wpływu zmiany parametrów takich jak: wysokość położenia środka masy i rozkład masy na osie na wartości tych granicznych sił wzdłużnych na kołach. W wyniku realizacji algorytmu DSM uzyskuje się charakterystyki w polu ograniczonym czworokątem i składające się z izolinii o stałych wartościach przyspieszeń poprzecznych osiąganych dla danych sił wzdłużnych na kołach przedniej i tylnej osi. Analizując otrzymane wykresy można stwierdzić, jaki wpływ na uzyskane wartości sił wzdłużnych na kołach jezdnych pojazdu ma zmiana danego parametru.

Słowa kluczowe: dynamika pojazdu, siły wzdłużne i poprzeczne, siły przyczepności, przyspieszenie wzdłużne i poprzeczne, Dynamic Square Method

1. Introduction

Passenger safety has long been a concern for vehicle manufacturers. In this regard, active safety, i.e. prevention of possible collisions and accidents has been of major importance. Accidents and collisions are frequent because the driver often overestimates the vehicle or does not adapt the driving style to the present road conditions. If the maximum friction force of the wheels is exceeded, the vehicle may lose its stability and divert from the direction intended by the driver. Such situation may result in a tragic accident.

Therefore the following question should be considered: what the potential achievable limit driving force (or braking force) values for the wheel to road contact point are under selected traffic conditions, and whether the curvilinear motion of the vehicle is possible, for the conditions. The correct answer to that question is of crucial meaning. This article attempts to answer this question, and presents the application of an algorithm that uses the Dynamic Square Method (DSM).

This method determines what maximum longitudinal forces can occur and whether it is still possible to transfer the lateral force to allow the curvilinear motion of the vehicle. A detailed description of this algorithm is included in [1] and this article presents its practical application. The analysis focused on the influence of parameters, such as the height of the center of gravity to wheelbase ratio, and the distribution of mass to individual axles on the limit force values to wheels. For this purpose, a simulation program based on the DSM algorithm was developed. The simulation testing was based on a two-wheel vehicle model.

2. A brief demonstration of the Dynamic Square Method

The Dynamic Square Method (DSM) is used to determine the force limit values on wheels in rectilinear and curvilinear motion of the vehicle. This method was first mentioned in 1995 and applied by M. Kato, K. Isoda and H. Yuasa [2]. Also M. Klomp [3, 4] referred to the method in his research papers. The DSM method has been thoroughly described by the author in [1]. The DSM method has also been used in the development of the active rear differential design for a Mitsubishi sports car model [7]. Also, the method is used to design and control active drive train systems [5, 6].

In order to utilize the DSM method, it is necessary to pre-set the longitudinal force values on the vehicle wheels. These would the basis for calculating the longitudinal acceleration and dynamic vertical loads acting on the front and rear axle wheels. This will facilitate the determination of the friction force for each axle. If this force is sufficient, it is possible to transfer the lateral force as well, which will allow the vehicle to move in curvilinear motion. On the other hand, if the attraction force value is lower than the assumed longitudinal force, the longitudinal force of such value will not be achieved.

The DSM algorithm enables the generation of diagrams for lateral accelerations (or lateral forces) dependent on the longitudinal forces on the vehicle's drive wheels.

For the analysis of the Dynamic Square Method a two-wheel vehicle model (Figure 2.1 and 2.2) was used.

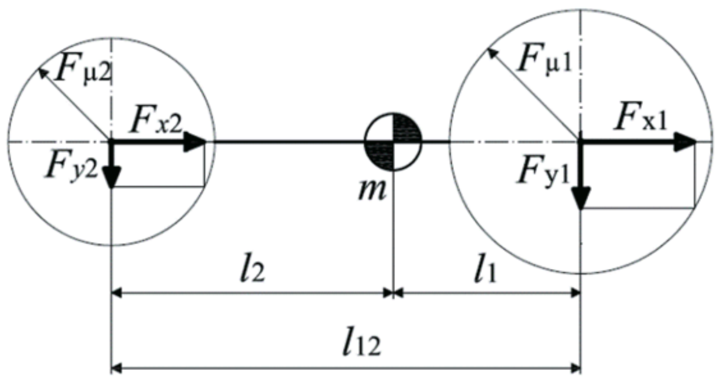

Fig. 2.1. The two-wheel vehicle model – top view

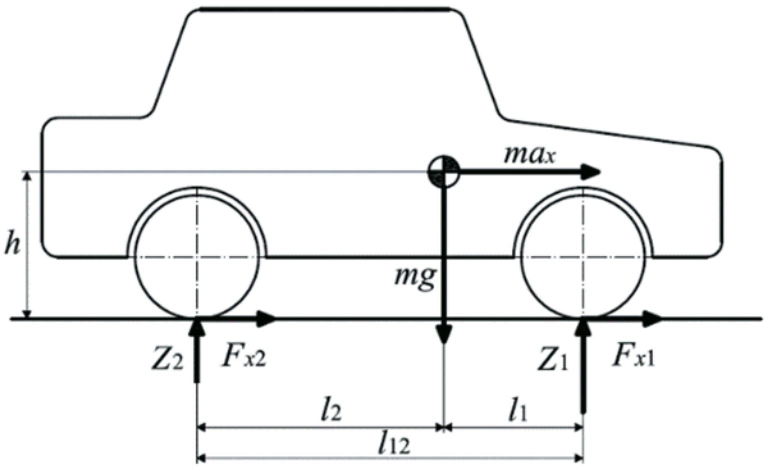

Fig. 2.2. The two-wheel vehicle model – side view

For the purposes of the used model an assumption of friction isotropy was made, i.e. that the wheel traction grip coefficient assumes the same value in all directions ($\mu_{xm} = \mu_{ym} = \mu_m$), with further assumption that the friction coefficient of the front μ_{m1} and rear μ_{m2} axle wheels is identical and equal to μ_m. The friction isotropy is illustrated by the friction circle (Fig. 2.3).

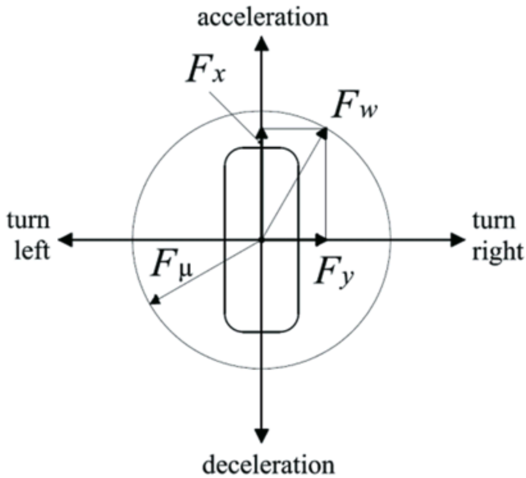

Fig. 2.3. Tire friction circle

On the basis of the tire friction circle (Figure 2.3), the lateral force value F_y may be inferred, according to the following relation:

$$F_y = \sqrt{F_\mu^2 - F_x^2} \tag{2.1}$$

From the calculated values of the longitudinal forces $F_{x1(2)}$ for both axles (index 1 – front axle, index 2 – rear axle) and the lateral forces on the wheels of individual axles $F_{y1(2)}$, the characteristics of the limit forces acting upon the wheels can be derived depending on the lateral acceleration a_y of the vehicle (See Figure 2.4). In order to represent the characteristics fully, the longitudinal a_x and lateral a_y acceleration values, resulting from the impact of the longitudinal $F_{x1(2)}$ and lateral $F_{y(1,2)}$ forces on the vehicle, must be known.

The longitudinal acceleration a_x is a result of the longitudinal forces acting on the wheels of a particular vehicle axle, and can be calculated using Newton's second law of dynamics:

$$a_x = \frac{\sum F_x}{m} = \frac{F_{x1} + F_{x2}}{m} \tag{2.2}$$

In contrast, the lateral acceleration of the vehicle a_y, occurring during the curvilinear motion, is calculated by the following relation[2]:

$$a_y = min\left(\frac{F_{y1}}{m_1}, \frac{F_{y2}}{m_2}\right) \tag{2.3}$$

The diagram of limit forces acting on wheels for the reference four-wheel drive vehicle was drawn up for the following characteristics, as shown in Table 2.1.

Table 2.1. Reference Vehicle Parameters

Vehicle mass m [kg]	1450
Mass per front axle m_1 [kg]	870
Mass per rear axle m_2 [kg]	580
Friction coefficient μ_m [-]	1.0
Wheelbase l_{12} [m]	2.65
Front axle distance from the center of mass l_1 [m]	1.06
Rear axle distance from the center of mass l_2 [m]	1.59
The height of the center of gravity h [m]	0.53

Table 2.2 shows the assumed value ranges of longitudinal forces on the wheels of the selected axis $F_{x1(2)}$. The positive values of the longitudinal forces on the $F_{x1(2)}$ wheels correspond to the driving forces on the $F_{n1(2)}$ wheels. On the other hand, the negative values of the longitudinal forces $F_{x1(2)}$ on the wheels represent the braking forces on the wheels $F_{h1(2)}$.

[2] The exact algorithm for determining the lateral acceleration is shown in [1].

Table 2.2. The assumed ranges for driving and braking forces for the reference vehicle

Driving force on front wheels F_{n1} [N]	(0; 12 000)
Braking force on front wheels F_{h1} [N]	(- 12 000; 0)
Driving force on rear wheels F_{n2} [N]	(0; 12 000)
Braking force on front wheels F_{h2} [N]	(-7000; 0)

Characteristics of limit forces on wheels obtained for the reference vehicle model (tables 2.1 and 2.2) is presented in the form of a graph in Figure 2.4.

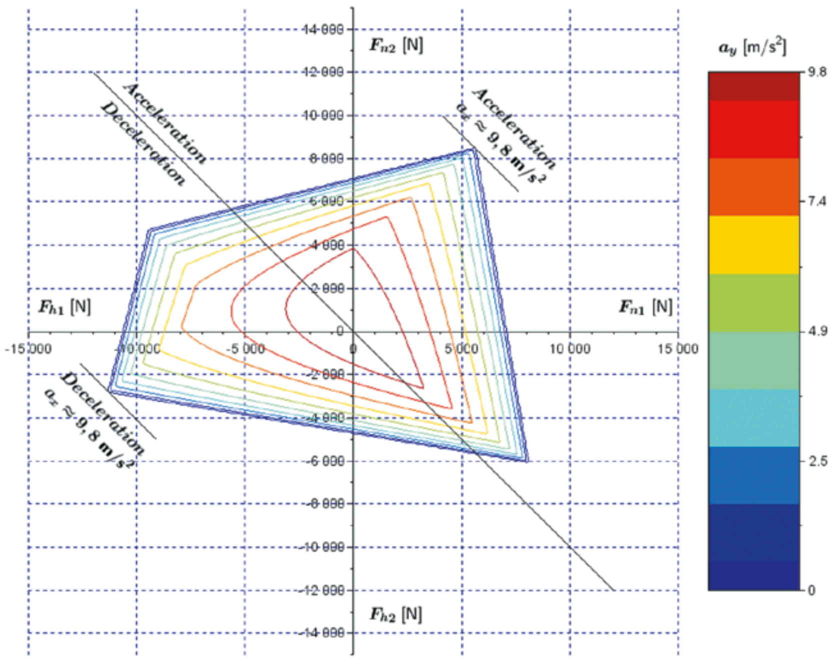

Fig. 2.4. The limit forces on wheels determined by the use of the DSM method for the reference vehicle (h/l_{12} = 0.2; μm = 1.0; m_1/m_2 = 1.5) [1]

Fig. 2.4 shows the dependence of the limit forces on the $F_{x1(2)}$ wheels and the lateral acceleration a_y of the vehicle. Maximum values of the longitudinal forces that can be transmitted by the front F_{x1} and rear F_{x2} wheels during acceleration and deceleration, both in rectilinear and curvilinear motion, can be estimated based on the obtained characteristics.

The characteristic obtained with the DSM (Figure 2.4), allows therefore to establish the maximum longitudinal forces acting on the vehicle wheels $F_{x1(2)}$, and the corresponding possible lateral acceleration a_y of the vehicle.

It is worth noting that only the upper section of the graph corresponding to the vehicle's acceleration and the left lower section of the graph corresponding to the vehicle braking are meaningful from the practical point of view. Conversely, the remaining parts of the characteristics (i.e. the upper left and lower right) are of no practical meaning.

The lines with a 45° inclination angle represent the fixed values of the sum of the longitudinal forces for both axles. The maximum total value of the driving forces on the axles is shown in the upper right section of the graph (Figure 2.4), and the maximum total value of the braking forces in the bottom right section of the graph. The force values at these points will be henceforth referred to as the maximum values of the driving or braking forces.

The maximum values of the aggregate driving force ($a_x \approx 9.8$ m/s^2) for a four-wheel drive vehicle or the maximum values of the aggregate braking force ($a_x \approx -9.8$ m/s^2) can only be achieved during the rectilinear motion of the vehicle ($a_y = 0$ m/s^2). Then, the entire friction force of the given axis $F_{\mu1(2)}$ is used for the driving force or braking force. Thus, the maximum achievable values of driving forces in rectilinear motion (see the upper right section of the characteristics in Figure 2.4) on the vehicle front and rear wheels are as follows: $F_{n1} \approx 5700$ N, $F_{n2} \approx 8500$ N. On the other hand, the braking force values on the front and rear wheels are respectively: $F_{h1} \approx -11\,400$ N and $F_{h2} \approx -2800$ N.

Using the DSM method, it is possible to observe the way the parameter changes affect the dynamics of a vehicle, including:

- the height of the center of gravity to wheelbase ratio h/l_{12},
- the mass distribution to front and rear wheels ratio m_1/m_2.

3. The influence of the height of the center of gravity to wheelbase ratio on the limit forces on drive wheels.

This section describes the influence of the height of the center of gravity to wheelbase ratio (h/l_{12}) on the limit forces on the drive wheels $F_{x1(2)}$.

Figures 3.1 and 3.2 show the limit forces characteristics on wheels $F_{x1(2)}$ for two different values of $h/l_{12} = 0.15$ and $h/l_{12} = 0.25$ obtained by the DSM method.

When analyzing Figures 3.1 ($h/l_{12} = 0.15$), 2.4 ($h/l_{12} = 0.2$) and 3.2 ($h/l_{12} = 0.25$) it is possible to evaluate the influence of the height of the center of gravity to wheelbase ratio (h/l_{12}) on the achieved longitudinal forces on front F_{x1} and rear F_{x2} wheels.

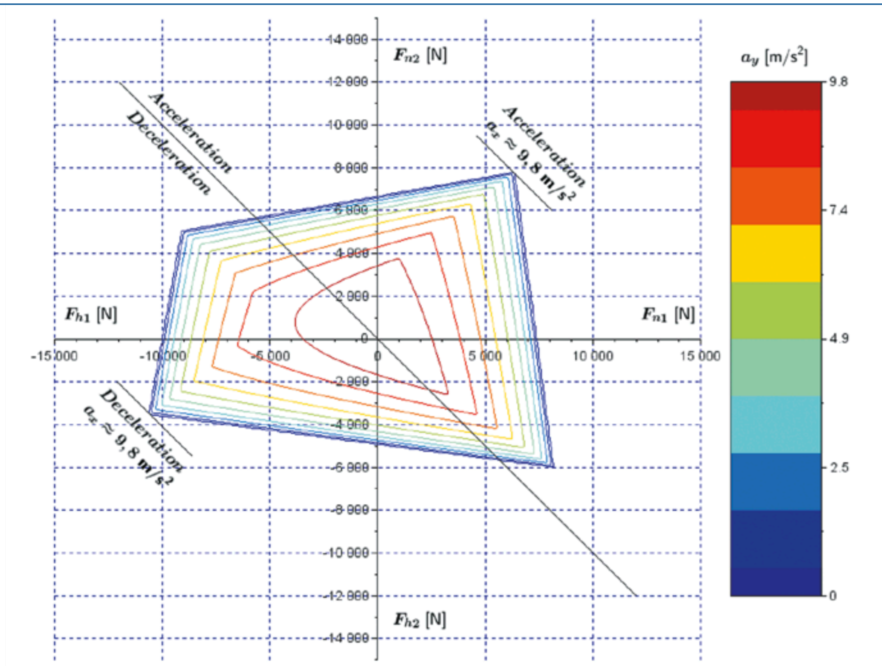

Fig. 3.1. The limit forces on wheels determined using the DSM method for h/l_{12} = 0.15 (μ_m = 1.0; m_1/m_2 = 15)

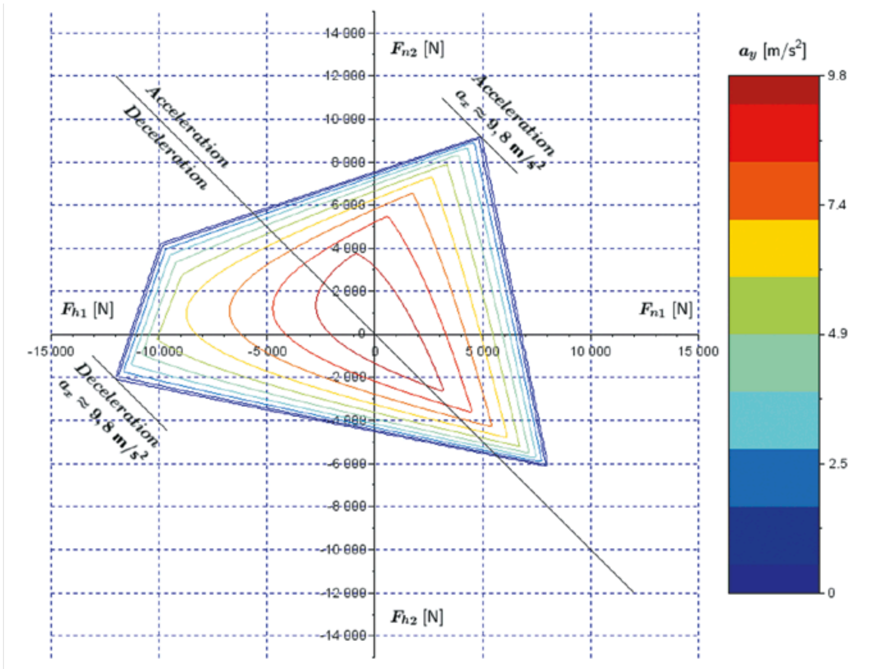

Fig. 3.2. The limit forces on wheels determined using the DSM method for h/l_{12} = 0.25 (μ_m = 1.0; m_1/m_2 = 1.5)

Decreasing the h/l_{12} ratio from 0.2 (the reference vehicle) to 0.15 results in a possibility of greater driving force ($F_{n1} \approx$ 6400 N) being transmitted during acceleration ($a_x \approx$ 9.8 m/s²) whilst travelling in rectilinear motion (see the right upper section of Fig. 3.1) than in the case of the reference vehicle (see Fig. 2.4). On the other hand, the driving force on the rear wheels ($F_{n2} \approx$ 7800 N) is decreased as compared to the reference vehicle (see Fig, 2.4). In addition the increase of the h/l_{12} ratio to 0.25 results in a possibility of lower driving force ($F_{n1} \approx$ 4900 N) being transmitted during acceleration ($a_x \approx$ 9.8 m/s²) whilst travelling in rectilinear motion (see the right upper section of Fig. 2.4) than in the case of the reference vehicle and the vehicle with the decreased h/l_{12} ratio (see the right upper section of Fig. 2.4, Fig. 3.2 and Fig. 3.2). Conversely, the rear wheel driving force ($F_{n2} \approx$ 9300 N) is higher than for the reference vehicle (see Fig. 2.4).

Decreasing the h/l_{12} ratio from 0.2 (reference vehicle) to 0.15 results in a possibility of a greater braking force ($F_{h2} \approx$ – 3500 N) being transmitted by the rear wheels of the vehicle during maximum braking of deceleration value ($a_x \approx$ 9.8 m/s²) whilst travelling in rectilinear motion (see the left bottom section of Fig. 3.1) than in the case of the reference vehicle (see Fig. 2.4). Conversely, the braking force on the front wheels ($F_{n2} \approx$ 10 700 N) is decreased as compared to the reference vehicle (see Fig. 2.4). Increasing the h/l_{12} ratio from 0.2 (reference vehicle) to 0.25 results in a possibility of a lower braking force ($F_{h2} \approx$ – 2000 N) being transmitted by the rear wheels of the vehicle during maximum braking of deceleration value ($a_x \approx$ 9.8 m/s²) whilst travelling in rectilinear motion (see the left bottom section of Fig. 3.2) than in the case of the reference vehicle (see Fig. 2.4). Here the braking force on the front wheels ($F_{h1} \approx$ - 12 200 N) is increased as compared to the reference vehicle (see Fig. 2.4).

4. The impact of the front to rear axle mass distribution ratio on the limit forces on the vehicle's drive wheels

This section will attempt to describe the impact of the front to rear axle mass distribution ratio (m_1/m_2) on the limit longitudinal force values on the vehicle's wheels $F_{x1(2)}$.

Figures 4.1 and 4.2 show the DSM based characteristics of limit forces on wheels $F_{x1(2)}$ for two different mass distributions on axles m_1/m_2 = 1.0 (50 % of the mass on front and rear axle each) and m_1/m_2 = 0.67 (40 % of the mass on the front axle and 60 % the rear axle).

By analyzing Figures 4.2 (m_1/m_2 = 0.67), (m_1/m_2 = 1.0) and 2.4 (m_1/m_2 = 1.5) it is possible to evaluate the influence of the mass distribution to the axles m_1/m_2 on the longitudinal forces achieved on front F_{x1} and rear F_{x2} wheels.

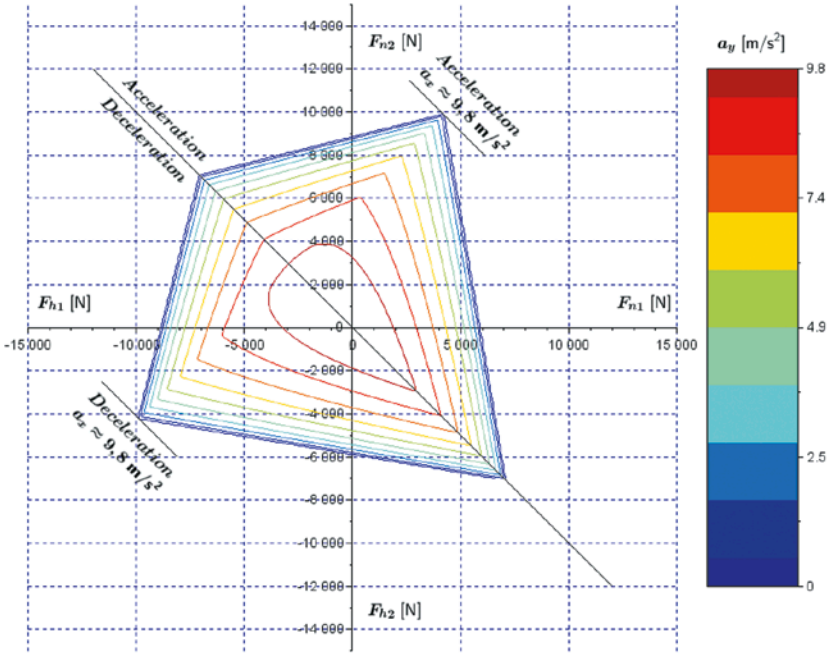

Fig. 4.1. The limit forces on wheels determined using the DSM method for the mass distribution to the axles of m_1/m_2 = 1.0 (h/l_{12} = 0.2, μ_m = 1.0)

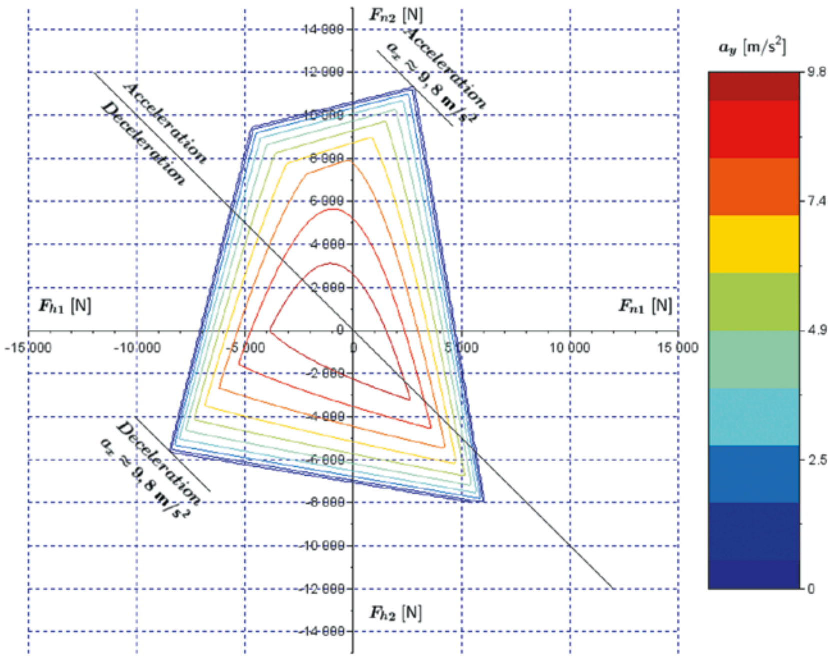

Fig. 4.2. The limit forces on wheels determined using the DSM method for the mass distribution to the axles of m_1/m_2 = 0.67 (h/l_{12} = 0.2, μ_m = 1.0)

Changing the mass distribution ratio for the front and rear axles from m_1/m_2 = 0.67 (reference vehicle) to m_1/m_2 = 1.0 results in the possibility of a decreased driving force ($F_{n1} \approx$ 4300 N) being transmitted during maximum acceleration ($a_x \approx$ 9.8 m/s^2) whilst travelling with rectilinear motion (see the right upper section of Fig. 4.1) by the front wheels, and an increased driving force ($F_{n2} \approx$ 9900 N) being transmitted by the rear wheels as compared to the reference vehicle (see Fig. 2.4). A similar situation occurs in the case of a vehicle with a mass ratio of m_1/m_2 = 0.67 and moving under the same conditions. In this situation the front wheels will transfer a decreased driving force ($F_{n1} \approx$ 3000 N), whilst the rear wheels will be able to transfer an increased force ($F_{n1} \approx$ 11 200 N) as compared to the reference vehicle (Figure 2.4).

During intensive braking with a deceleration of $a_x \approx$ 9.8 m/s^2 (see the left bottom section of Fig. 4.1) the change of the front to rear axle mass distribution ratio from m_1/m_2 = 1,5 (reference vehicle) to m_1/m_2 = 1.0 results in the ability of the rear wheels of a vehicle in rectilinear motion to achieve a higher braking force ($F_{h2} \approx$ – 4300 N) as compared to the reference vehicle (see Fig. 2.4). On the other hand, the decreased static mass on the front axle results in a decreased braking force ($F_{h1} \approx$ – 9900 N) being transferred by the wheels of this axle as compared to the reference vehicle (Figure 2.4). For the mass distribution of m_1/m_2 = 0.67 for a vehicle rectilinear motion with the maximum achievable deceleration under the given conditions ($a_x \approx$ 9.8 m/s^2) can achieve the following maximum braking forces: $F_{h1} \approx$ – 9400 N for the front wheels and $F_{h2} \approx$ – 4800 N for the rear wheels (see the bottom right corner of Fig. 4.2). As compared to the reference vehicle (see Fig. 2.4), the braking force on the front wheels is lower, while on the rear wheels it is higher than for the reference vehicle.

5. Conclusions

The article has discussed the use of the Dynamic Square Method (DSM) to analyze the impact of selected vehicle parameters on the limit forces on vehicle's drive wheels.

The analysis of limit forces on wheels has been carried out, whilst taking into consideration the change of selected parameters, including the height of the center of gravity to wheelbase ratio and the distribution of mass for individual axles. Based on the analysis of the presented examples, the DSM is useful for analyzing the longitudinal forces $F_{x1(2)}$, i.e., the driving $F_{n1(2)}$ and braking $F_{h1(2)}$ forces, depending on the lateral acceleration values a_y, both in rectilinear and curvilinear motion.

The increase of the height of the center of gravity to wheelbase ratio increases the driving force on the rear axle wheels and reduces the driving force on the front axle wheels as compared to the reference vehicle. The value of the front axle braking force is greater than for the reference vehicle, whilst the braking force of the rear wheels is lower than for the reference vehicle.

The decrease in the height of the center of gravity to wheelbase ratio decreases the driving force on the rear axle wheels and increases it on the front axle wheels as compared to the reference vehicle. The value of the front axle wheels braking force is lower than for the reference vehicle, whilst the braking force value of the rear wheels is greater than for the reference vehicle.

The decrease in the front to rear wheels mass distribution ratio decreases the driving force on the front axle wheels and increases it on the rear axle wheels as compared to the reference vehicle. The value of the front axle wheels braking force is lower than for the reference vehicle, whilst the braking force value of the rear wheels is greater than for the reference vehicle.

The full text of the article is available in Polish online on the website http://archiwummotoryzacji.pl.

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